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Math Club Teaser #33

October 30, 2009
(due November 6, 2009)

Solution

We use the letters a, b, c, f, g for the numbers of **a**dults, **b**oys, **c**hildren, **f**amilies, and **g**irls respectively. The first restriction of the problem says that $c > a > b > g > f$. Then $a \geq f + 3$ because a is at least one more than b which is at least one more than g which is at least one more than f. Since $a = 2f$, we get $2f \geq f + 3$, and we found that $\boxed{f \geq 3}$

In the same manner, we see that $b \leq a - 1$ and $g \leq a - 2$. Adding these two inequalities, and remembering that $a = 2f$, gives $\boxed{c \leq 4f - 3}$

Now we can show that too many families would mean more children than the restrictions allow. See, the smallest family has at least one child; the next has at least two (each family has a different number of children); the next at least three, and so on. Also, the last family has at least one more than all the others combined, so $c \geq \big(1 + 2 + \ldots + (f-1)\big) + \big(1 + 2 + \ldots + (f-1) + 1\big) = f^2 - f + 1$. With the inequality in the second box, this says $f^2 - f + 1 \leq 4f - 3$; i.e., $f^2 - 5f + 4 = (f-1)(f-4) \leq 0$. In other words, $\boxed{f \leq 4}$

If f was 4, there would be $1 + 2 + 3 + 7 = 13$ children (no more because $c \leq 4 \cdot 4 - 3 = 13$). But $a = 8$, so in order to reach $c = 13$, we would need $b = 7$ and $g = 6$. The single child would be boy (every girl has a brother), so the other families would have two girls each (no three girls can be sisters). But the family with two girls is impossible because they must have a brother! It must be that $\boxed{\boxed{f = 3}}$, so $a = 6$, and $\boxed{\boxed{b = 5\ , g = 4}}$ (according to the first restriction).

Solved by:

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