

I U P U I
MATH CLUB TEASER #46

September 10, 2010
(due September 17, 2010)

SOLUTION

Say the left jar has the fewest marbles, with W_ℓ white and T_ℓ total marbles, so that $T_\ell \leq 20$. The probability of drawing a white marble is

$$\frac{1}{2} \left(\frac{W_\ell}{T_\ell} + \frac{20 - W_\ell}{40 - T_\ell} \right),$$

(remember that both jars are equally likely to be picked.) Now move a white marble from the right jar to the left, and a black marble from the left to the right. In particular, the number of marbles in each jar has not changed, but now the probability of drawing a white marble is

$$\frac{1}{2} \left(\frac{W_\ell + 1}{T_\ell} + \frac{19 - W_\ell}{40 - T_\ell} \right).$$

The difference between these probabilities is $\frac{1}{2} \left(\frac{-1}{T_\ell} + \frac{1}{40 - T_\ell} \right) \leq 0$ because $T_\ell \leq 20$. This means that the marble transfer improved the odds. After transferring as many times as we can, the left jar will contain only white marbles, or the right jar will contain only black ones! The best situation is when there is only one white marble in the left jar, and the rest of the marbles in the right jar. The probability of drawing a white marble from that configuration is

$$\frac{1}{2} \left(1 + \frac{19}{39} \right) \approx 0.743589\dots$$

SOLVED BY:

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