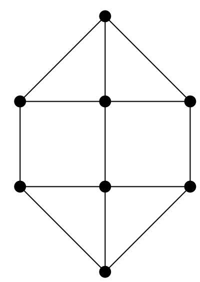
I U P U I
Math Club Teaser #59

March 31, 2011
(due April 8, 2011)

Solution

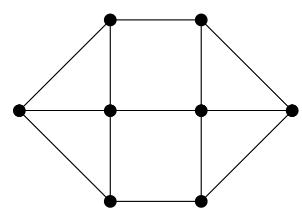
The probability of being able to cross to the other side is $\frac{1}{2}$, but this is not as easy as it seems.

First we draw a graph that represents the bridge configuration. Put a vertex for every land region, and connect two vertices whenever there is a bridge.
The graph looks like that $\longrightarrow$



Now look at the problem from the viewpoint of a boat moving between the eight water regions delineated by the bridges. The boat can move between adjacent regions exactly when the corresponding bridge is down.

If we draw the graph that represents the connections between water regions we find that it looks exactly like the previous graph:



Since the two graphs are equal, every bridge configuration that connects the two river banks is matched with a bridge configuration that lets a boat cross from the left to the right. But the boat can cross exactly when the river banks are not connected and viceversa.
This means that for every bridge configuration that lets the Professor cross, there is a corresponding configuration that does not. Since their numbers are equal, the probability of being able to croos the river is $\frac{1}{2}$.

<u>Solved by:</u>

`Captain Nemo.`